Resummation of non-global logs at finite Nc

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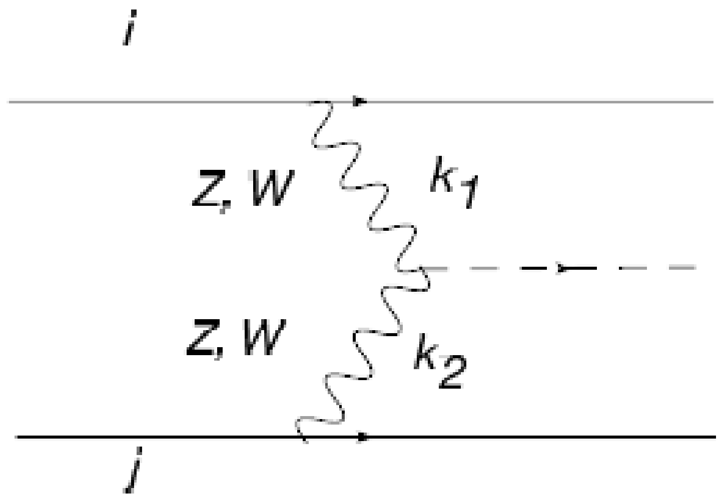
Outline

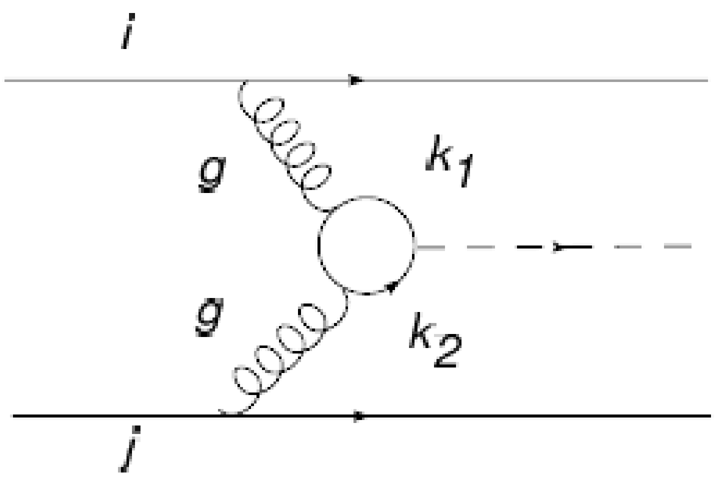
- Motivation : Energy flow at the LHC
- Non-global logarithms
- Relation to saturation physics
- Weigert's approach and its refinement
- Numerical result

Energy flow at the LHC

Higgs plus di-jet events



Vector boson fusion



Gluon fusion

Different patterns of soft gluon radiation
Could be used to extract Higgs couplings, suppress large backgrounds

Cox, Forshaw, Pilkington; Englert, Spannowsky, Takeuchi,...

Cross section with a jet veto

Require that no jets with transverse momentum greater than p_t^{veto} is produced in $gg \to H$

Double logarithms $\quad (\alpha_s L^2)^n \quad L = \ln \frac{m_H}{p_t^{veto}}$

→ exponentiate

$$\exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right)$$

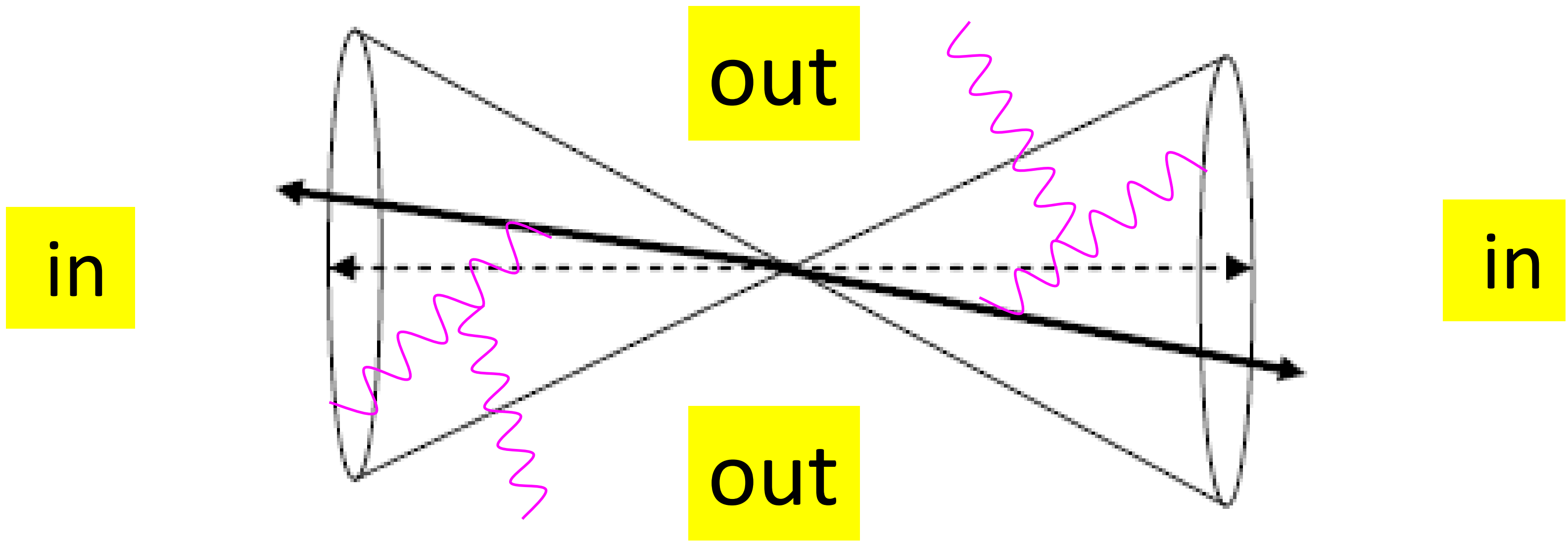
LL $\quad$ NLL $\quad$ NNLL

Banfi, Salam, Zanderighi; Becher, Neubert

Note: the observable is global,
i.e. all particles in the final state are measured.

Non-global observables

Measurement is done only in a part of the phase space excluding jets



Gluons are emitted at large angle, resum only the soft logarithms

Do not exponentiate,
Resummation done only at large-Nc

Dasgupta, Salam (2001)

Banfi-Marchesini-Smye (BMS) equation

Probability that energy flow into the ``out" region is less than p_t^{veto}

$$\partial_\tau P_{\alpha\beta} = N_c \int \frac{d\Omega_\gamma}{4\pi} \mathcal{M}_{\alpha\beta}(\gamma) \Big(\Theta_{in}(\gamma) P_{\alpha\gamma} P_{\gamma\beta} - P_{\alpha\beta} \Big)$$

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})}$$

$$\tau = \frac{\alpha_s}{\pi} \ln \frac{p_t}{p_t^{veto}}$$

Equation derived at large-Nc

Generalization to finite Nc → Weigert (2003)

BK and JIMWLK equations

Small-x evolution for the dipole S-matrix

BK

$$\partial_\tau \langle S_{xy}\rangle_\tau = N_c \int \frac{d^2z}{2\pi} \mathcal{M}_{xy}(z) \Big(\underline{\langle S_{xz}\rangle_\tau \langle S_{zy}\rangle_\tau} - \langle S_{xy}\rangle_\tau \Big)$$

$$\mathcal{M}_{xy}(z) = \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$

$$\tau \equiv \frac{\alpha_s}{\pi} \ln \frac{1}{x}$$

finite Nc

JIMWLK

$$\partial_\tau \langle S_{xy}\rangle_\tau = N_c \int \frac{d^2z}{2\pi} \mathcal{M}_{xy}(z) \Big(\underline{\langle S_{xz} S_{zy}\rangle_\tau} - \langle S_{xy}\rangle_\tau \Big)$$

Solving the JIMWLK equation

Operator form : $\partial_\tau \langle S_{xy} \rangle_\tau = -\langle \hat{H} S_{xy} \rangle_\tau$

$$\hat{H} = \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{K}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y\right)^{ab} \nabla_y^b$$

Can be viewed as a Fokker-Planck equation

Solve the associated Langevin equation

Blaizot-Iancu-Weigert: Rummukainen, Weigert

For this purpose, it is important that the kernel

$$\mathcal{K}_{xy}(z) = \frac{(x-z)\cdot(z-y)}{(x-z)^2(z-y)^2}$$ is factorized

Weigert's approach

Finite-Nc generalization of BMS in operator form

$$\partial_\tau \langle P_{\alpha\beta}\rangle_\tau = -\langle \hat{H} P_{\alpha\beta}\rangle$$

$$\hat{H} = \frac{1}{2}\int d\Omega_\alpha d\Omega_\beta \frac{d\Omega_\gamma}{4\pi} \underline{\mathcal{M}_{\alpha\beta}(\gamma)} \nabla^a_\alpha \left(1 + \tilde{U}^\dagger_\alpha \tilde{U}_\beta - \Theta_{in}(\gamma)\left(\tilde{U}^\dagger_\alpha \tilde{U}_\gamma + \tilde{U}^\dagger_\gamma \tilde{U}_\beta\right)\right)^{ab} \nabla^b_\beta$$

The kernel is factorized

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})} = \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k_\gamma)(k_\gamma \cdot p_\beta)}$$

→ Langevin equation

A flaw in the argument

$$\mathcal{M}_{\alpha\beta}(\gamma) = \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k_\gamma)(k_\gamma \cdot p_\beta)}$$

The kernel is indeed factorized ….but in four-momentum space

``Gaussian” noise

$$\langle \xi_a^{(I)\mu} \xi_b^{(J)\nu} \rangle \sim \delta_{ab} \delta^{IJ} \underline{g^{\mu\nu}}$$

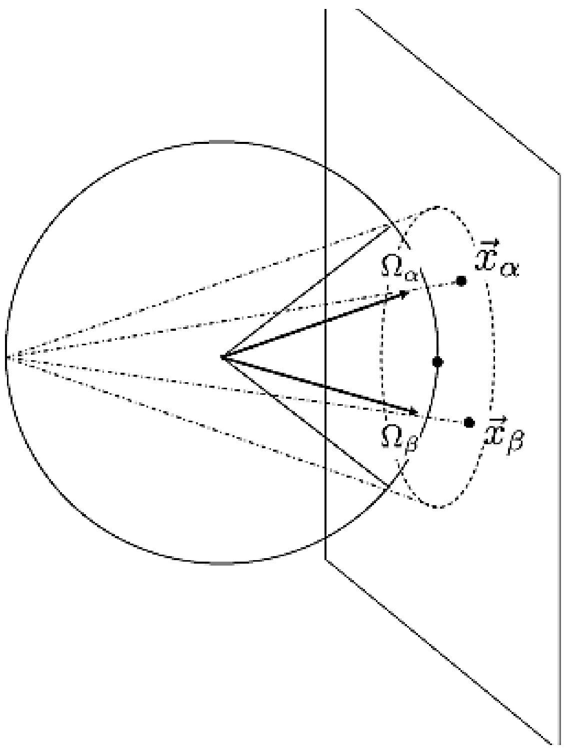
Not positive definite

Promoting similarity to equivalence

Stereographic projection exactly maps the two physics

YH (2008)

$$\frac{d^2 z}{2\pi}\frac{(x-y)^2}{(x-z)^2(z-y)^2} = \frac{d\Omega_\gamma}{4\pi}\frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})}$$



True also in the strong coupling limit of N=4 supersymmetric Yang-Mills

Alternative JIMWLK Hamiltonian

YH, Iancu, Itakura, McLerran (2004)

$$\hat{H} = \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{K}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y\right)^{ab} \nabla_y^b$$

$\Updownarrow$ equivalent

$$\hat{H} = \frac{1}{2} \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{M}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y\right)^{ab} \nabla_y^b$$

$$\mathcal{K}_{xy}(z) = \frac{(x-z)\cdot(z-y)}{(x-z)^2(z-y)^2} \qquad \mathcal{M}_{xy}(z) = \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$

Effective kernel in jet physics

YH & Ueda

BK/JIMWLK

$$\mathcal{M}_{xy}(z) = \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$

$$\mathcal{K}_{xy}(z) = \frac{(x-z)\cdot(z-y)}{(x-z)^2(z-y)^2}$$

BMS

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})}$$

$$\boxed{\mathcal{K}_{\alpha\beta}(\gamma) = \frac{(n_\alpha - n_\gamma)\cdot(n_\gamma - n_\beta)}{2(1-n_\alpha\cdot n_\gamma)(1-n_\gamma\cdot n_\beta)}}$$

New !

factorized
in 3D Euclidean metric

The Langevin equation

$$U_\alpha(\tau+\varepsilon) = e^{iA^L_\alpha} U_\alpha(\tau) e^{iA^R_\alpha}$$

$$A^L_\alpha = \sqrt{\frac{\varepsilon}{4\pi}} \int d\Omega_\gamma \frac{(n_\alpha - n_\gamma)^k}{1 - n_\alpha \cdot n_\gamma} \left(-\Theta_{in}(\gamma) U_\gamma t^a U^\dagger_\gamma \xi^{(1)k}_{\gamma a} + \Theta_{out}(\gamma)\, t^a \xi^{(2)k}_{\gamma a}\right)$$

$$A^R_\alpha = \sqrt{\frac{\varepsilon}{4\pi}} \int d\Omega_\gamma \frac{(n_\alpha - n_\gamma)^k}{1 - n_\alpha \cdot n_\gamma} t^a \xi^{(1)k}_{\gamma a}.$$

noise

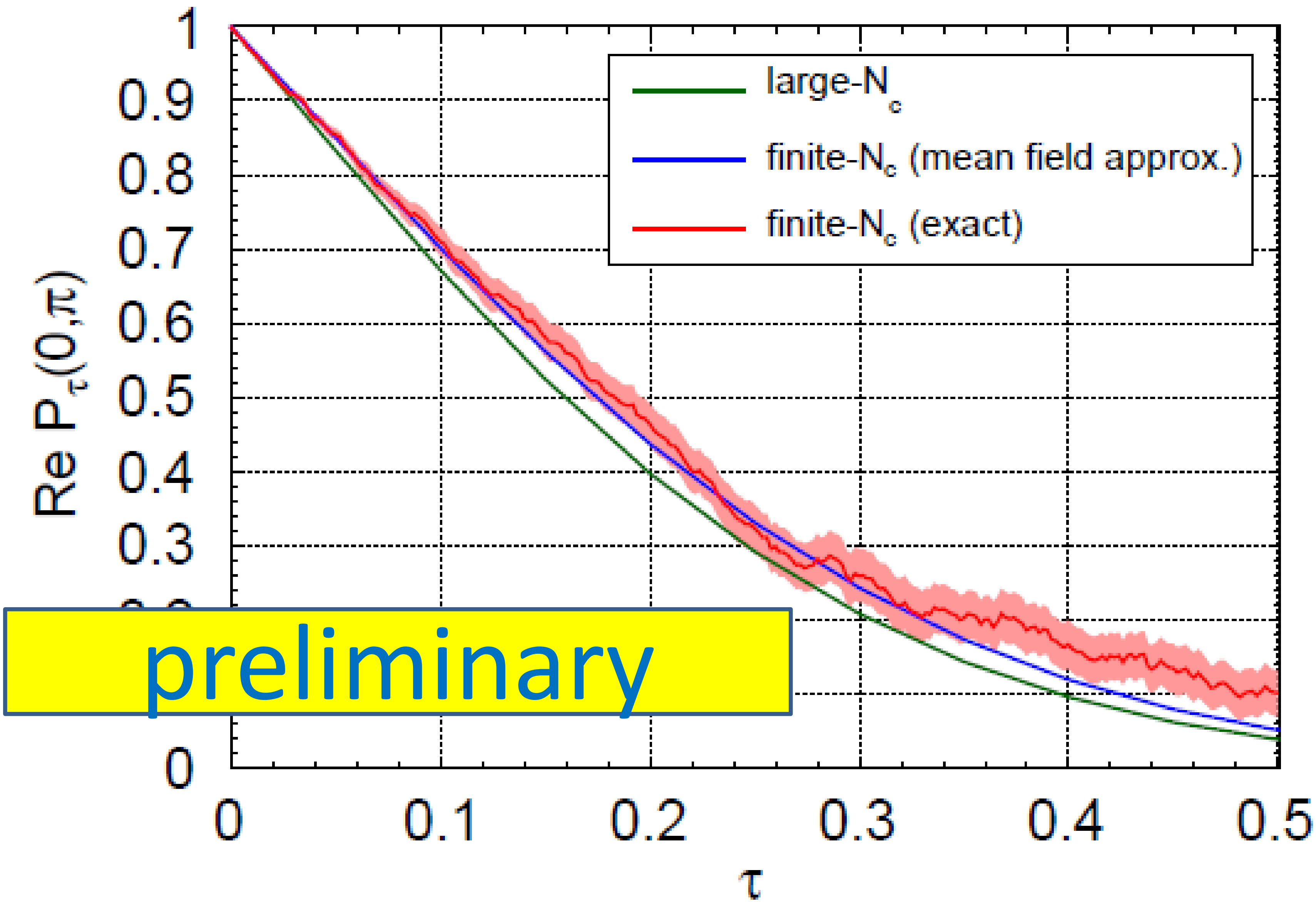
Calculate the average

$$\frac{1}{N_c} \mathrm{tr}(U_\alpha(\tau) U^\dagger_\beta(\tau))$$

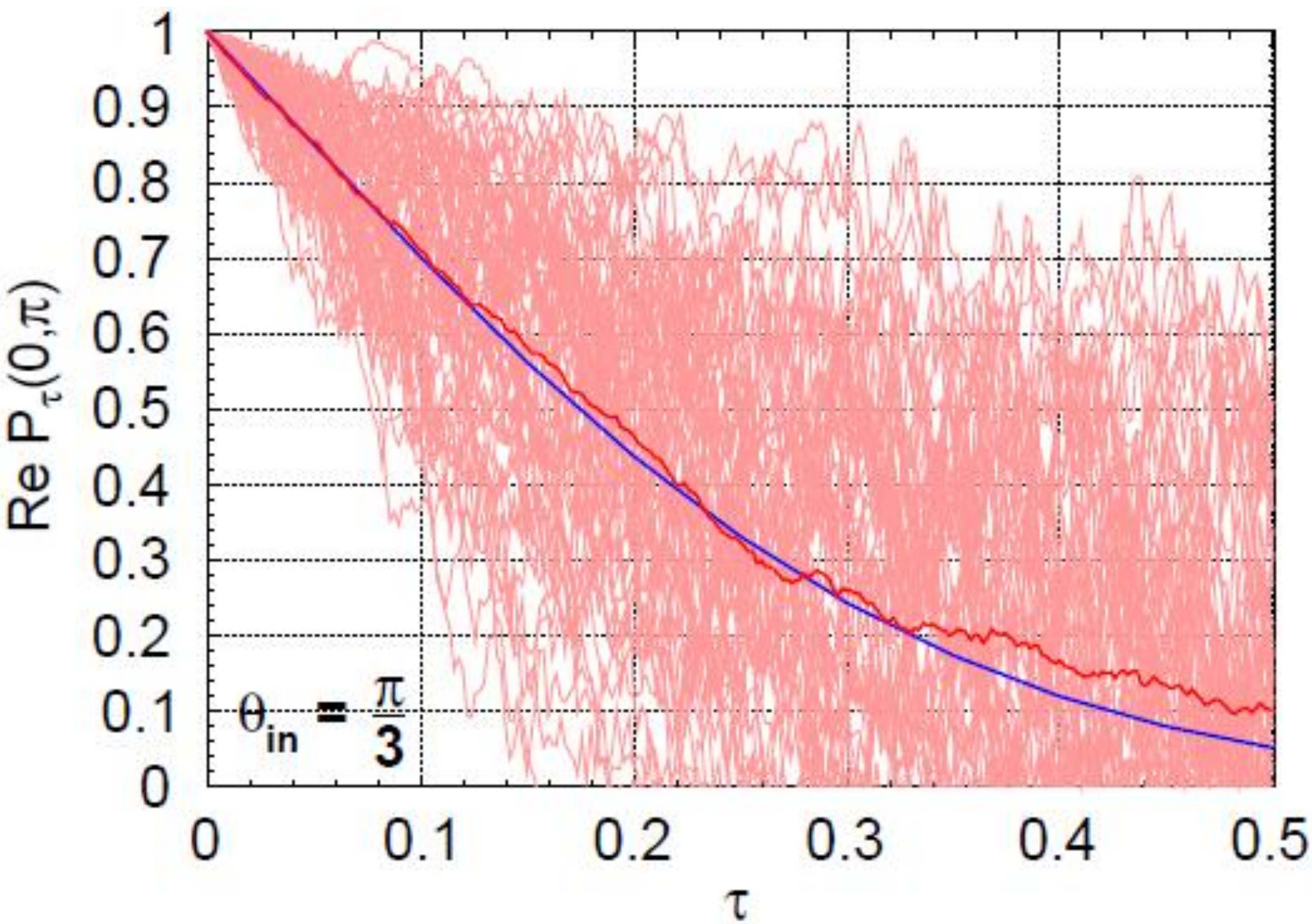
over many random walk trajectories

Result

Back-to-back jets,
65 random walks



Fluctuation of random walks



Summary and outlook

- First quantitative result of the resummation of non-global logs at finite Nc.
- Fluctuation very big. Mean field approximation violated. The initial condition matters.

YH & Mueller; Avsar & YH.

- Extension to hadron collisions